EGC220 Class Notes 2/14/2023

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$$F = \underbrace{\bar{A}BC + \bar{A}B\bar{C}} + AC$$

$$\bar{A}B(\underbrace{C+\bar{C}}_{1})$$

$$\underbrace{}$$

$$\bar{A}B + AC$$

TABLE 2-6
Basic Identities of Boolean Algebra

1. $X + 0 = X$	2. $X \cdot 1 = X$	
3. $X + 1 = 1$	4. $X \cdot 0 = 0$	
5. $X + X = X$	6. $X \cdot X = X$	
7. $X + \overline{X} = 1$	8. $X \cdot \overline{X} = 0$	
9. $\overline{\overline{X}} = X$		
10. $X + Y = Y + X$	11. $XY = YX$	Commutative
12. $X + (Y + Z) = (X + Y) + Z$	13. $X(YZ) = (XY)Z$	Associative
14. $X(Y + Z) = XY + XZ$	15. $X + YZ = (X + Y)(X + Z)$	Distributive
16. $\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17. $\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

$$XY + X\overline{Y}$$
$$= X\underbrace{(Y + \overline{Y})}_{1}$$
$$= X$$

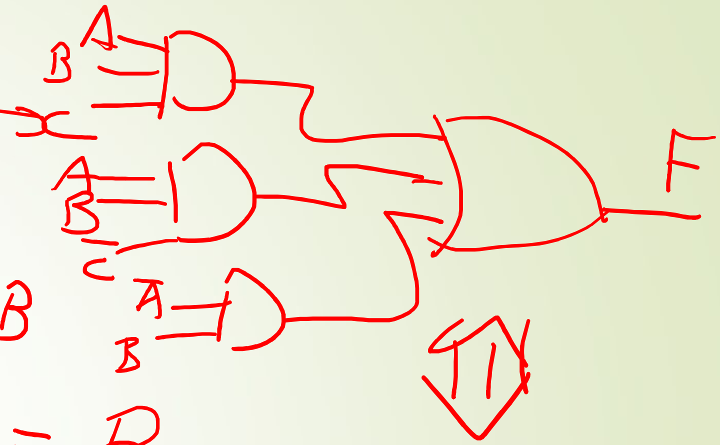
$$\overline{C} + XC = (\overline{C} + X)\underbrace{(\overline{C} + C)}_{1}$$

1. Simplify the following Boolean expression as much as possible.

a. ABC + A'B +ABC'

b. (X+Y)'(X'+Y')

c. XY + X(WZ +WZ')



a. 14 3

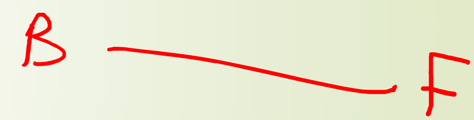
$AB(C+\bar{C}) + \bar{A}B = AB + \bar{A}B$

2 7 1

$= B(A+\bar{A}) = B$

14

7 1 2



b.

$\overline{(X+Y)}(\bar{X}+\bar{Y})$

$\bar{X}\bar{Y} \rightarrow A$

16 $\bar{X}\bar{Y}(\bar{X}+\bar{Y}) = \bar{X}\bar{X}\bar{Y} + \bar{X}\bar{Y}\bar{Y} = \bar{X}\bar{Y} + \bar{X}\bar{X} = \bar{X}' \rightarrow \overline{(X+Y)}$

14 6 5

c. 14 7 2

$XY + X(WZ + W\bar{Z}) = XY + XW$

$W(Z+\bar{Z})$

2. Without simplifying, find the dual of

a. A’C’ + ABC + AC’

$(\bar{A}+\bar{C})(A+B+C)(A+\bar{C})$

b. A’B(D’+C’D) + B(A + A’CD)

$(\underbrace{\bar{A}B(\bar{D}+\bar{C}D)}_{\text{Dual}})(\underbrace{B(A+\bar{A}CD}_{\text{Dual}})$

$(\underbrace{AB}_{\text{Dual}}+(\underbrace{\bar{D}+\bar{C}D}_{\text{Dual}}))(B+(\underbrace{A+\bar{A}CD}_{\text{Dual}}))$

$=(\bar{A}+B+\bar{D}(\bar{C}+D))$
$(B+A(\bar{A}+C+D))$

3. Without simplifying, find the complement of

$\bar{F} =$ a. A'C' + ABC + AC'

$$F = (\overline{\bar{A}\bar{C}})(\overline{ABC})(\overline{A\bar{C}})$$

$$= (A+C)(\bar{A}+\bar{B}+\bar{C})(\bar{A}+C)$$

b. A'B(D'+C'D) + B(A + A'CD)

$$\bar{F} = \overline{\bar{A}B(\bar{D}+\bar{C}D) + B(A+\bar{A}CD)}$$

$$= \left(\overline{\bar{A}B(\bar{D}+\bar{C}D)}\right)\left(\overline{B(A+\bar{A}CD)}\right)$$

$$= \left(\overline{\bar{A}B} + (\overline{\bar{D}+\bar{C}D})\right)\left(\bar{B} + (\overline{A+\bar{A}CD})\right)$$

$$= \left(A+\bar{B} + D(C+\bar{D})\right)\left(\bar{B} + \bar{A}(A+\bar{C}+\bar{D})\right)$$

4. Reduce the following Boolean expression to the indicated number of literals:

a. A'C' + ABC + AC' to three literals

b. A'B(D'+C'D) + B(A + A'CD) to one literal

(14) (7) (2)

a. $\bar C(\bar A + A) + ABC$

$\bar C + ABC$

$= \bar C + AB$

(15) $X + YZ = (X+Y)(X+Z)$

$\underbrace{\bar C}_{X} + \underbrace{AB}_{Y}\underbrace{C}_{Z} = (\bar C + AB)(\underbrace{\bar C + C}_{1})$

b. $\bar A B(\overset{X}{\bar D} + \overset{Y}{\bar C}\overset{Z}{D}) + B(\overset{X}{A} + \overset{Y}{\bar A}\overset{Z}{CD})$

$\bar A B(\bar D + \bar C)\underbrace{(\bar D + D)}_{1} + B\underbrace{(A + \bar A)}_{1}(A + CD)$

$= \bar A B(\bar D + \bar C) + B(A + CD)$

$= \bar A B \bar D + \bar A B \bar C + AB + BCD$

$= \bar A B \bar D + B(\bar A \bar C + A) + BCD$

$= \bar A B \bar D + B(\cancel{\bar A + A})(\bar C + A) + BCD$

$= \bar A B \bar D + AB + B\bar C + BCD$

$= B(\bar A \bar D + A) + B(\bar C + CD)$

$= B\underbrace{(A + \bar A)}_{1}(\bar D + A) + B\underbrace{(\bar C + C)}_{1}(\bar C + D)$

$= B(\bar D + A) + B(\bar C + D)$

$$= B\bar{D} + AB + B\bar{C} + BD$$

$$B(\bar{D} + D)$$

$$= B + AB + B\bar{C}$$

$$= B(\underbrace{1 + A + \bar{C}}_{1})$$

$$= \underline{\underline{B}}$$

4, Reduce the following Boolean expression to the indicated number of literals:

A'B(D'+C'D) + B(A + A'CD) to one literal

different approach.

$$= \bar{A}B\bar{D} + \bar{A}B\bar{C}D + AB + \bar{A}BCD$$

$$= \bar{A}B(\underbrace{\bar{D} + \bar{C}D}) +$$

$$(\bar{D}+\bar{C})(\underbrace{\bar{D}+D}_{1})$$

$$= \bar{A}B\bar{C} + \bar{A}B\bar{D} + AB + \bar{A}BCD$$

$$= B(\bar{A}\bar{C} + A) + \bar{A}B(\bar{D}+CD) \Rightarrow$$

$$\cancel{AB+B\bar{D}}$$

$$B(A+\cancel{\bar{A}})(A+\bar{C})$$

$$+\bar{A}B(\bar{D}+C)(\cancel{\bar{D}+D})$$

$$= AB + B\bar{C} + \bar{A}B\bar{D}$$

$$+\bar{A}BC$$

$$B(A+\bar{A}\bar{D}+\bar{C}+\bar{A}C)$$

$$\Leftarrow B[(\cancel{A+\bar{A}})(A+\bar{D}) + (\bar{C}+\bar{A})(\cancel{\bar{C}+C})]$$

$$\Rightarrow B(\underbrace{A+\bar{D}+\bar{C}+\bar{A}}_{1}) = B$$

5. Find he complement of F = XY+Z’. Then show that FF’ = 0 and F+F’ =1

$$G = \overline{F}$$

$$\overline{F} = \overline{XY + \overline{Z}}$$

$$\overline{XY} \cdot \overline{\overline{Z}}$$

$$(\overline{X} + \overline{Y})Z$$

$$F\overline{F} = 0$$

$$(XY + \overline{Z})(\overline{X}Z + \overline{Y}Z)$$

$$\underbrace{XY\overline{X}Z}_{0} + \underbrace{XY\overline{Y}Z}_{0} + \underbrace{\overline{Z}\overline{X}Z}_{0} + \underbrace{\overline{Z}\overline{Y}Z}_{0}$$

$$= 0$$

$$F + \overline{F} = 1$$

$$\overline{X}Z + \overline{Y}Z + XY + \overline{Z} \quad (\overline{X} + \overline{Z})(\underbrace{Z + \overline{Z}}_{1}) + \overline{Y}Z$$

$$= \overline{X} + \overline{Z} + \overline{Y}Z + XY = \underbrace{(\overline{X} + X)}_{1}(\overline{X} + Y) + (\overline{Z} + \overline{Y})\underbrace{(\overline{Z} + Z)}_{1}$$

$$= \overline{X} + Y + \overline{Z} + \overline{Y} = \overline{X} + 1 + \overline{Z} = 1$$

6. For function F = XY + XY' + Y'Z

a. List the truth table.

b. If possible, simplify the function further.

c. Draw an AND OR implementation of the function.

$\Rightarrow X(Y+\bar{Y}) + \bar{Y}Z$

$X + \bar{Y}Z$

a.

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

b. $F = \bar{X}\bar{Y}Z + \underbrace{X\bar{Y}\bar{Z} + X\bar{Y}Z}_{X\bar{Y}(\bar{Z}+Z) \downarrow X\bar{Y}} + \underbrace{XY\bar{Z} + XYZ}_{XY(\bar{Z}+Z) \downarrow XY}$

$X(\bar{Y}+Y) \downarrow$

$\bar{X}\bar{Y}Z + X = \underbrace{(\bar{X}+X)}_{1}(\bar{Y}Z+X)$

$= \bar{Y}Z + X$

c.

7, For the following Boolean expression $F = XY + X'Y'Z' + X'YZ'$, determine

a. Truth table
b. Sum of min terms
c. Product of max terms
d. Standard sum of products

4	2	1	
X	Y	Z	F
0	0	0	1 $\rightarrow m_0$
0	0	1	0
0	1	0	1 $\rightarrow m_2$
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1 $\rightarrow m_6$
1	1	1	1 $\rightarrow m_7$

b. $F = \Sigma m(0,2,6,7)$

c. $F = \Pi M(1,3,4,5)$

d. $F = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + XY\bar{Z} + XYZ$

7. For the following Boolean expression F = XY + X'Y'Z' + X'YZ', determine

a. Truth table

b. Sum of min terms

c. Product of max terms

d. Standard sum of products

e. standard product of sums

f. min sum of prod.

a.

X (4)	Y (2)	Z (1)	F	$\bar{F}$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

b. $F = \Sigma m(0,2,6,7)$

c. $F = \Pi M(1,3,4,5)$

d. $F = \underbrace{\bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z}}_{\bar{X}\bar{Z}(\bar{Y}+Y)} + \underbrace{XY\bar{Z} + XYZ}_{XY(\bar{Z}+Z)}$

$= \bar{X}\bar{Z} + XY$

e. $\bar{F} = \Sigma m(1,3,4,5)$

$\bar{F} = \bar{X}\bar{Y}Z + \bar{X}YZ + X\bar{Y}\bar{Z} + X\bar{Y}Z$

$F = (\overline{\bar{X}\bar{Y}Z})(\overline{\bar{X}YZ})(\overline{X\bar{Y}\bar{Z}})(\overline{X\bar{Y}Z})$

$= (X+Y+\bar{Z})(X+\bar{Y}+\bar{Z})(\bar{X}+Y+Z)(\bar{X}+Y+\bar{Z})$

8. For G = F’ of problem 7, determine

a. Truth table
b. Sum of min terms
c. Product of max terms
d. Standard sum of products

a.

A	B	C	F	G
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

b. $\cancel{F} = G = \Sigma m(1,3,4,5)$

c. $G = \Pi M(0,2,6,7)$

d. $F = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C$